JOURNAL

OF THE

WASHINGTON ACADEMY OF SCIENCES

VOL. I, NOVEMBER 4, 1911. No. 7

APPLIED MATHEMATICS.—*A criterion for best magnitudes in precise measurement.* MAYO DYER HERSEY, Bureau of Standards. Communicated by L. A. Fischer.

In the determination of a quantity y from observations upon the independent quantities $x_1, x_2, \ldots x_n$ with which y is connected by the relation

$$y = f(x_1, x_2, \ldots x_n)$$

there is frequently an option as to the order of magnitude which shall be assigned to the x's before setting up apparatus. The problem of best magnitudes has long been recognized as a fundamental one in the theory of the precision of measurements, but available discussions are limited in their usefulness by the assumption of constant errors or proceed by a step-by-step calculation invented anew for each problem.[1]

The present analysis aims to formulate the reasoning once for all, setting up a general criterion from which special cases may be derived at will.

This is accomplished by the consideration of two functions ϕ and Θ beside the function f characterizing the instrument or method. If in general Δx denote an error in the observed quan-

[1] See, for example, Holman, "Precision of Measurements," pp. 100–119; Mellor, "Higher Mathematics for Students of Chemistry and Physics," pp. 531–540; Baum, "General Method for Determining Best Reading of an Instrument for Least Error," Phys. Rev. 1899: pp. 181–182.

tity x, then the function $\phi(x)$ is defined by the statement $\Delta x = \phi(x) \cdot \epsilon$, where the constant ϵ represents this error at some designated region of x, such as the zero point of a scale. On the other hand, if Δy be the resultant error, then the function $\Theta(y)$ is defined by the understanding that it is $\Theta(y) \cdot \Delta y$ that we wish to make a minimum by our choice of best magnitudes for the x's.

Granting that the resultant error is

$$\Delta y = \sum_{r=1}^{r=n} \frac{\partial y}{\partial x_r} \cdot \Delta x_r$$

it is seen that to make $\Theta(y) \cdot \Delta y$ a minimum, requires a minimum for

$$\Theta(f) \cdot \sum_{r=1}^{r=n} \frac{\partial}{\partial x_r} f(x_r) \cdot \phi(x_r) \cdot \epsilon_r$$

Differentiating by any particular x such as x_k will lead to the following condition for a maximum or minimum if we simplify the notation by omitting functional parentheses and by letting p stand for $\partial f / \partial x$, and if we cast out as physically zero all derivatives of ϕ_r by x_k except that of ϕ_k itself:

$$\frac{1}{\Theta} \frac{\partial \Theta}{\partial x_k} \sum_{r=1}^{r=n} \epsilon_r \phi_r p_r + \epsilon_k \frac{\partial \phi_k}{\partial x_k} p_k + \sum_{r=1}^{r=n} \epsilon_r \phi_r \frac{\partial p_r}{\partial x_k} = 0 \ldots\ldots (1)$$

This equation is the type for a series of n simultaneous equations, the group constituting the criterion.

Those who are partial to the theory of probability will remark that we have taken for Δy the actual error $\sum \frac{\partial y}{\partial x} \Delta x$ and not the (smaller) probable error $\left[\sum \left(\frac{\partial y}{\partial x} \Delta x\right)^2\right]^{\frac{1}{2}}$. The treatment of best magnitudes differs, however, from the greater part of the theory of errors in that it deals with the prevention and not with the computation of errors, and therefore must concern itself with the case when the errors do reinforce each other by pure addition. Nevertheless, the criterion for minimum probable error can be obtained from the other by replacing ϵ, ϕ, and p by ϵ^2, ϕ^2, and p^2, and multiplying the first summation by 2. When there is but

one independent variable to consider, the distinction between actual and probable error vanishes.

Now from (1) by substituting $n = 2$, $x_1 = x$, $x_2 = z$, $p_1 = p$, $p_2 = q$, $\Theta = 1/f$, and $\phi = 1$, we have the important special case of two independent variables with constant errors ϵ_x and ϵ_z, a minimum fractional error $\Delta y/y$ being desired in the result. The criterion reduces to the single equation

$$(p - q) \cdot R = \left(\frac{\partial R}{\partial x} - \frac{\partial R}{\partial z}\right) \cdot f \quad \ldots\ldots\ldots\ldots (2)$$

where R denotes $(p\epsilon_x + q\epsilon_z)$.

Finally for one independent variable, if we denote differentiation by primes and put $p = f'$, (1) becomes:

$$f' \phi \Theta' + f' \phi' \Theta + f'' \phi \Theta = 0 \quad \ldots\ldots\ldots\ldots (3)$$

This breaks up into eight special cases upon assigning to Θ the fundamental values 1 and $1/f$ while ϕ is restricted to 1 and x. The four most important in practice are the following:

1. When seeking a minimum absolute error, $\Theta = 1$, consequently the criterion becomes:

$$f' \phi' + f'' \phi = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

2. When seeking a minimum fractional error $\Theta = 1/f$.

$$\therefore \frac{\phi'}{\phi} = \frac{f'}{f} - \frac{f''}{f'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

3. When seeking minimum absolute error with constant error in x, $\Theta = 1$ and $\phi = 1$, therefore

$$f'' = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

4. When seeking minimum fractional error with constant error in x, $\Theta = 1/f$ and $\phi = 1$, so that

$$f' = \sqrt{f f''} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

To fix in mind the meaning of the criterion, two examples, in themselves sufficiently simple to be verified by trial, may be added,—

I. Find best resistance for heating coil if power input $y = x \cdot z$ is deduced from readings of 15 a. ammeter with error ϵ_x and

150 v. voltmeter with error $\epsilon_z = 10\epsilon_x$. By (2), $p = \partial/\partial x(xz) = z$, while $q = x$; also $R = z\epsilon_x + x\epsilon_z$; $\partial R/\partial x = \epsilon_z$, $\partial R/\partial z = \epsilon_x$. Substituting and reducing,

$$\frac{z}{x} = \sqrt{\frac{\epsilon_z}{\epsilon_x}} = 3.2 \text{ ohms.}$$

The condition for minimum probable error is $z/x = 4.6$ ohms.

II. Find best order of interference x in determining relative wave-length $y = C/x$, if an error of observation of 1/100 wave-length at zero path-difference becomes $\frac{1}{100}\left(1 + \frac{2x^2}{10^9}\right)$ on separating plates. Using (5), with $f' = \frac{-C}{x^2}$, $f'' = 2C/x^3$, and $\phi' = 4x/10^9$ gives at once the result $x = 22{,}000$.

In practice a distinction must be drawn between precision and accuracy.[2] If Δx denote the precision of observation of x, the criterion leads to the best attainable precision in y; but if, from a knowledge of the reliability of his method, the investigator can go further and employ for Δx an estimate of accuracy, the formulae lead to the conditions for best accuracy in y.

In either event, it is the aim of the criterion to afford rules of interest to the designer and experimenter rather than to the computer.

For their interest and criticism acknowledgment is gratefully made to President Woodward of the Carnegie Institution and to Drs. Burgess, Dorsey, Buckingham and Mr. L. A. Fischer of the Bureau of Standards.

RADIO-TELEGRAPHY.—*Notes from the U. S. Naval Wireless Telegraphic Laboratory.* L. W. AUSTIN.

1. Wave Length and Ground Absorption of Electrical Waves

Since the early days of wireless telegraphy, the country lying to the north and north-east of Newport, Rhode Island, has been known to show a very large absorption for electrical waves.

[2] In short, accuracy = (reliability of method) $\pm$ (precision of observation).

During the experiments in long distance radiotelegraphy[1] carried on at Brant Rock during the summer of 1910, the signals sent out by the Birmingham lying at Newport were measured at Brant Rock 45 miles away. The wave lengths used were 1000 meters and 3750 meters and the received signals were measured on a 15 ohm tellurium-constantan thermoelement.[2] One millimeter on the thermoelement galvanometer corresponded to a current of 263×10^{-6} amperes.

The data of the observations are shown in the table. Under the column Calculated, are given the values taken from Table XVI of the paper cited. The difference between the calculated and observed values represents the excess of the ground absorption over that which would have been observed if the waves had passed over salt water.

BIRMINGHAM AT NEWPORT, RECEIVED AT BRANT ROCK

WAVE LENGTH	SENDING ANTENNA CURRENT	DEFLECTION THERMOELEMENT	RECEIVED ANTENNA CURRENT 10^{-6} AMP.		ABSORPTION OF SIGNALS
Meters	*Amperes*	*Millimeters*	*Observed*	*Calculated*	*per cent*
1000	28.7	16.0	1050	5400	95
3750	26.3	37.5	1600	1550	0

If we consider the strength of signal which is proportional to I^2, it will be found that 95 per cent of the energy was absorbed in the case of the 1000 meter wave, while in the case of the 3750 meter wave the ground absorption was not detectable, the observed signals being slightly stronger than the calculated values.

The experiment explains fully why with the short wave lengths ordinarily used, wireless communication has been so difficult over this region and indicates that where such difficulties are encountered longer wave lengths should probably be employed.

[1] Bureau of Standards Bulletin, 7: 315. 1911. Abstract, this Journal, 1: 82. 1911.

[2] See paper cited, p. 317.

2. Directive Action of the Marconi Bent Antenna at Clifden, Ireland

While the directive action of the Marconi bent antenna at short distances has been amply demonstrated by Marconi, Fleming and others, it has not seemed certain that this radiation asymmetry would not diminish as the distance increased so that at great distances the radiation distribution would be the same as that of an ordinary antenna.

Recently at the request of the wireless laboratory, the National Electric Signaling Company made telephone shunt measurements on the signals received at their Brant Rock station from the Marconi transatlantic station at Clifden. These indicated an average received energy in the day time of 5×10^{-8} watts or in terms of current through 25 ohms as used in Table XVI of my paper already cited, 45×10^{-6} amperes. I have been unable to obtain from the Marconi Company any data on the value of the antenna current at Clifden, but nevertheless some conclusions can be drawn from the information at hand. The distance from Clifden to Brant Rock is 2640 nautical miles. The height of the Clifden antenna considered as a flat top is approximately 200 feet, the effective receiving height of the Brant Rock umbrella antenna was found in the recent long distance experiments to be approximately 350 feet. The wave length of Clifden is a little more than 6000 meters and the total available power is 1100 h.p.[3]

In Tables XVI and XVII already mentioned, $h_1 = 200$ feet, $h_2 = 350$ feet, $d = 2500$ miles, in round numbers, and $I_r = 45 \times 10^{-6}$ amperes. We may calculate from the tables that the sending antenna current $I_s = 810$ amperes which is an enormous value. If the total antenna resistance is only 1 ohm it corresponds to 660 K.W. which is more than the whole 1100 h.p. would be able to transmit to the antenna. But this resistance is improbably small.

From this it must be concluded that unless the formula is grossly in error the bent Clifden antenna shows a decided directive effect even at this distance.

[3] Fleming, Principles of Electric Wave Telegraphy.

3. *A Table of Radiation Resistances for Various Wave Lengths and Antenna Heights*

The radiation theory of Hertz shows that the radiated energy of an oscillator may be represented by

$$E = \text{const. } h^2/\lambda^2 I^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where h is the length of the oscillator, λ the wave length and I the current at its center.

A Table of Antenna Radiation Resistances

(h = height to center of capacity of conducting system)

h= / WAVE LENGTH λ	40 ft.	60 ft.	80 ft.	100 ft.	120 ft.	160 ft.
m	*ohm*	*ohm*	*ohm*	*ohm*	*ohm*	*ohm*
200	6.0	13.4	24.0	37.0	54.0	95.0
300	2.7	6.0	10.6	16.5	23.8	42.4
400	1.5	3.4	6.0	9.3	13.4	23.8
600	0.66	1.5	2.7	4.1	6.0	10.6
800	0.37	0.84	1.5	2.3	3.4	6.0
1000	0.24	0.54	0.95	1.5	2.1	3.8
1200	0.17	0.37	0.66	1.03	1.5	2.6
1500	0.106	0.24	0.42	0.66	0.95	1.7
2000		0.134	0.24	0.37	0.54	0.95
2500			0.15	0.24	0.34	0.61
3000			0.106	0.17	0.24	0.42
4000			0.060	0.093	0.134	0.24

h= / λ	200 ft.	250 ft.	300 ft.	450 ft.	600 ft.	1200 ft.
m	*ohm*	*ohm*	*ohm*	*ohm*	*ohm*	*ohm*
600	16.4	25.8	37.4	84.0	149.0	
800	9.2	14.5	21.0	47.0	84.0	
1000	6.0	9.3	13.5	30.0	54.0	215.0
1200	4.1	6.5	9.3	21.0	37.0	149.0
1500	2.6	4.1	6.0	13.4	24.0	95.0
2000	1.5	2.3	3.4	7.5	13.4	54.0
2500	0.95	1.49	2.2	4.8	8.6	34.0
3000	0.66	1.03	1.5	3.4	6.0	24.0
4000	0.37	0.58	0.84	1.9	3.4	13.4
5000	0.24	0.37	0.53	1.20	2.2	8.6
6000	0.16	0.26	0.37	0.84	1.49	6.0
7000	0.121	0.19	0.27	0.61	1.09	4.4

It has been shown by Ruedenberg[4] that for a flat top antenna the constant in the above formula is in round numbers 1600, or

$$E = 1600\, h^2/\lambda^2 I^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The term $1600\, h^2/\lambda^2$ is commonly spoken of as the radiation resistance, inasmuch as it is a quantity which when multiplied by the square of the current gives the radiated energy.

The accuracy of equation (1) was verified in the experiments on long distance radio-telegraphy already cited. It has therefore seemed worth while to calculate a table of the radiation resistance for various heights and wave lengths.

ELECTRICITY.—*Methods for the measurement of the effective inductance of resistance coils.* F. W. GROVER and H. L. CURTIS. Communicated by E. B. Rosa. To appear in the Bulletin of the Bureau of Standards.

In the measurement of inductances which are associated with relatively high resistances, such as resistance coils, the multipliers of voltmeters and wattmeters and the like, the ordinary bridge methods using alternating current of suitable frequency are sufficiently sensitive, but they fail to give an accurate determination due to the fact that the inductance of each arm of the bridge is of the same magnitude as the inductance to be measured. The method which we have used to overcome this difficulty is to substitute for the unknown inductance, an inductance of approximately the same resistance, but of such a form that its inductance can be computed from its dimensions. From the changes in the bridge which it was necessary to make to restore a baiance, the difference in the inductance of the two resistances was computed.

Three bridge methods were used. The first was Maxwell's method of comparing an inductance with a capacity; the second was Anderson's modification of the above; the third was the use of a variable inductance in one of the arms. These methods gave concordant results and the method to be chosen in any given case will largely depend upon the value of the resistance associated with the inductance which is being measured.

[4] R. Ruedenberg, Ann. d. Phys. 25: 446. 1908.

The standards used were in the form either of parallel wires or of circles. Only in the case of one-ohm coils was it practicable to use both forms. The agreement between the two forms was satisfactory as is shown by the following example. The difference in the computed values, after allowing for the inductance of the binding posts, was 128 x 10^{-9} henry. The measured difference in the inductance was also 128 x 10^{-9} henry. With standards having larger resistance different sizes and lengths of wire were sometimes used, as well as different distances apart. As an example, the 5000-ohm parallel wire standard was measured with the wires 2 cm. apart and again with them 10 cm. apart. The observed difference was 114 microhenrys. The computed difference when taking into account the distributed capacity between the wires was 110 microhenrys.

While in all cases the standards were adjusted to have nearly the same value of resistance as the coil to be measured, yet it was necessary to be able to vary the resistance thru a small range without varying the inductance. This was accomplished by two different devices. Steps of a tenth of an ohm were obtained by substituting for manganin links, copper links of the same size and form. A continuous variation of resistance from 00 to 0.1 ohm was obtained by means of a copper wire sliding in a tube of mercury. The change in inductance was very small in this case, but where small inductances were being measured, it was necessary to make a correction on this account.

The chief difficulty encountered was with coils of high resistance where the capacity between the wires appreciably affects the effective inductance. If L is the computed inductance, C the capacity between the wires and R their resistance, then the effective inductance L' is approximately given by the equation

$$L' = L - \tfrac{1}{3} \cdot CR^2.$$

In commercial resistance coils having resistances of 100 ohms or over the last term is usually the larger of the two, so that the effective inductance is negative. With coils greater than 1000 ohms the actual inductance is usually a negligible part of the effective inductance, so that the value of the effective inductance depends entirely upon the capacity and resistance. With parallel

wires the capacity not only depends upon the relative positions of the two wires, but upon their potential with respect to the earth. The capacity as usually computed requires that the potential of one wire shall be as much above earth potential as the other is below earth potential. To make sure that this requirement is fulfilled in practice, a special net work is arranged in which there are two indicating instruments, both of which must show zero current. Altho somewhat complicated, this method gives satisfactory results in those cases where the capacity to earth is an important factor.

The methods here outlined have been found to give satisfactory results with resistances from 1 to 10,000 ohms. With coils the resistance of which lies between 1 and 1000 ohms it was possible to measure the inductance with such accuracy that the error in the time constant (L/R) deduced from this measurement is not more than 10^{-9} second. For coils with a resistance of more than 1000 ohms the corresponding limit is 10^{-8} second.

ORGANIC CHEMISTRY.—*A new bromine method for the determination of aromatic phenols. Its special application to thymol.* ATHERTON SEIDELL, Hygienic Laboratory. Communicated by Walter Wyman.

A quantitative method for the determination of thymol being desired for use in experiments upon the solubility of this compound in various solvents, a search of the literature was made for a suitable method. Those found were the Messinger and Vortmann iodine method (Ber. **23**: 2753. 1890), the various adaptations of the bromine method with Koppeschaar's bromate-bromide solution, and the diazonitraniline method of Riegler (Buletinal soc. sciinte. Bucuresci, **8**: 51. 1899). Experiments with the first two of these gave unsatisfactory results and the sources of the errors were carefully investigated. In the case of the iodine method an absolutely definite ratio of thymol to alkali was found to be required in order that accurate results might be obtained, therefore necessitating the knowledge of the amount of thymol before beginning the determination. The difficulties which were encountered with the various adaptations of the bromine method appeared to be due to the too energetic action of the bromine in

aqueous solutions, or to its loss by volatilization during the determination. After many experiments these difficulties were surmounted and the following new method devised.

Briefly the principal involved is the conversion of the thymol to the dibrom compound, with formation of two molecules of hydrobromic acid, by addition of an excess of bromine vapor to an aqueous solution of the thymol in contact with a small amount of carbon bisulphide. After allowing to stand at ordinary temperature a short time, the excess of bromine is removed by means of potassium iodide and thiosulphate, and the free hydrobromic acid is then titrated after addition of excess of potassium iodate, by means of standard thiosulphate solution.

In practice the determination of thymol is made as follows. The weighed sample of 0.1 to 0.5 grams is placed in a 200 cc. glass stoppered bottle with 5 cc. CS_2 and about 100 cc. H_2O. Bromine vapor is then poured into the mixture until the color after thorough shaking shows that a considerable excess of Br is present. After one-half hour an excess of 10 per cent KI solution is added and the liberated iodine corresponding to the free bromine is titrated with 0.1 n thiosulphate, an excess of about 5 per cent KIO_3 solution is then added, the liberated iodine equivalent to the free HBr is titrated with 0.1 n thiosulphate. The difference between the two readings of thiosulphate gives the amount of HBr formed during the reaction, and calculating two molecules of HBr for each molecule of thymol the amount of the latter is directly obtained. A number of determinations gave the following results:

SAMPLE OF THYMOL	CUBIC CENTIMETERS STANDARD THIOSULPHATE		DIFFERENCE	CALCULATED PER CENT THYMOL
	Free Br.	Free HBr.		
0.3124	25.4	63.6	38.2	99.9
0.2229	34.8	62.0	27.2	99.7
0.2838	26.9	61.65	34.75	100.0
0.3915	15.7	63.7	48.0	100.3
0.2833	19.5	54.15	34.65	100.0
0.3257	24.9	64.5	39.6	99.4
0.4298	30.7	83.5	52.8	100.3

(Factors for standard thiosulphate, Readings x 1.086 = 0.1 N exactly).

These results show that even with considerable variation in the amount of excess of bromine used, no appreciable effect upon the results is obtained. The determinations can be easily made in less than an hour and no other standard solution than thiosulphate is required. The effect of the carbon bisulphide is to prevent the too vigorous action of the bromine on the thymol; without its use high results are obtained unless only the slightest excess of Br is used and in that case the results are apt to be too low on account of using insufficient Br to complete the reaction. The CS_2 appears therefore to act as a safety valve and assures accurate results even with considerable variation in the other conditions of the determination. The CS_2 also has the advantage that since it is an excellent solvent for thymol it can be used to extract this compound from its mixtures or solutions, and then by adding water and excess of Br the determination readily made as above outlined.

There is little doubt that this method is of general applicability to practically all aromatic phenols for the determination of which various other bromine methods have so far been suggested. It is my intention to ascertain the limits of the applicability of this method at an early date and publish the results in a more detailed paper elsewhere.

MARINE ENGINEERING.—*Thrust balancing in direct-connected, marine steam turbines.* E. BUCKINGHAM, Bureau of Standards. Communicated by C. W. Waidner.

It is desirable that in a direct-connected marine steam turbine, the steam thrust shall balance the propeller thrust so as to relieve the main thrust bearing, altho this bearing must be able to take up the full unbalanced thrust in either direction which may be thrown on it for short periods during maneuvering or as a result of accident. If such a balance could be attained only at the expense of considerable sacrifices in other directions or only for one particular speed of the ship, it would not be worth much attention, and it appears from published designs that many builders attach little importance to close balancing. It will be pointed out in this paper that a balance may be attained without

any important sacrifice of other qualities of the turbine, and furthermore that if the steam and propeller thrusts are balanced at full speed the balance will be maintained approximately at much lower speeds,—a matter of more consequence in naval than in commercial practice.

It may be regarded as a certainty that the standard direct-connected installation of the immediate future will have turbines of the mixed wheel-and-drum type driving independent shafts, each unit being complete in itself.

The operative advantages of the independent shaft system are obvious; and the superiority of the mixed drum-and-wheel construction is such that it is fast superseding the pure drum turbine, while the pure multicellular turbine has seldom been used and is now obsolete in marine practice. The drum construction is more compact than the wheel construction and permits of the use of more blade rows in a prescribed length, beside reducing windage. It should be used at the low pressure (L.P.) end and should extend toward the high pressure (H.P.) end as far as the steam volume and blade lengths are great enough that ample radial clearances can be used without serious leakage losses. Above this point, the use of impulse-wheel stages contracts the leakage ring to the diameter of a shaft bushing and permits of unlimited radial clearances. At the H.P. end the use of impulse stages with variable partial admission offers several well recognized advantages. The following remarks, therefore, refer only to the mixed turbine, and they relate to the problem of deciding where the drum should begin.

With a single rotor and unidirectional flow aft, the steam thrust, due in the main to the difference of pressure on the ends of the drum, is available for balancing the propeller thrust. From the speed-power curve of the hull the thrust may be found for the assigned speed at which a balance is desired; and from the estimated propulsive coefficient the shaft horse-power of the turbine computed. If the terminal conditions of pressure, superheat, and vacuum are specified, together with the revolutions per minute, we may then, from the data on efficiency obtained by experiment on stages of the general style to be used,

make a sketch design and determine approximately the form of the curve of pressure distribution thru the turbine at the given power. It is then easy, if the diameters of drum and shaft have been fixed, to decide what pressure should act on the H.P. end of the drum in order that steam and propeller thrusts shall be balanced. The pressure drop to this point is to be taken upon wheels, and the remainder on the drum. If the data are good and the computations correctly made, the rotor and shaft will float at some speed close to that assumed.

In general, it will be found that the length of the drum computed in this way is much less than would seem advisable if the question of thrust-balancing were not considered— there will be too many wheel stages and the efficiency will suffer because not enough blade-rows can be got into the prescribed length. With a turbine of large diameter, a satisfactory compromise is impossible. The writer has seen one design of turbines for a battleship, in which the drum was long enough that at full power the steam thrust was about 60 per cent greater than the propeller thrust, while the drum was still much shorter than was desirable from the point of view of economy.

The obvious escape from the conflict of requirements is the adoption of some double-flow scheme. This complication may not be at all worth while in small machines, but for large powers when a division of the turbine into two lighter parts is advantageous in itself, the double-flow arrangement offers a simple solution of the difficulty. Let the H.P. rotor consist of the wheel stages and the first drum while the L.P. rotor consists of a second drum—combined of course with the backing turbine. The steam flows aft thru the H.P. turbine, thence to the after end of the L. P. drum, and exhausts at the forward end of the L.P. casing. The two drums thus exert opposing thrusts and no matter what the pressure at the beginning of the H.P. drum, the intermediate pressure may be so chosen as to give the resultant steam thrust due to both drums any value we please. We are thus able to design the arrangement of stages as seems most desirable, regardless of thrust, using a long drum and only a few wheels. We then split the drum at the right point on the

pressure curve, turn the L.P. part end for end, and put in a separate casing. In this way we may get as close an approximation to exact balancing as is permitted by the accuracy of the data available in designing. The writer is not aware that this expedient has been used in marine turbine construction, but it is so obvious that it seems highly probable that it will be used soon.

The question next arises as to how nearly a balance attained for one speed will be preserved at other speeds, and it turns out that if the turbine is regulated in the ordinary way, a balance for full speed will be fairly maintained at much lower speeds. The amount of the residual thrust, forward or aft, at any speed, depends on various elements; and to compute it exactly, data corresponding to those already used for full speed and power must be available, but it will suffice here to treat a single case roughly.

Let us assume that at half speed the effective horse power needed to drive the ship is $\frac{1}{10}$ of that at full speed,—as it will be approximately. The propeller thrust is then $\frac{1}{8}$ of that at full speed, and if the thrust was balanced at full speed it will still be so if all the steam pressures are reduced to $\frac{1}{8}$ of their values at full power. We have, therefore, to ask how nearly this condition regarding the pressures is fulfilled.

The water rate of the turbine at half speed will probably be some 40 per cent greater than at full speed. Leaving out of account the change in propulsive efficiency, which is not great, we may conclude that at half speed the total amount of steam flowing thru the turbine will be about $1.4 \times \frac{1}{10}$ or $\frac{1}{7}$ nearly. How, then, is the pressure distribution in the turbine altered when the steam flow is cut down to $\frac{1}{7}$? The answer to this question is most opportune: if the steam flow is cut down, by throttling or by closing first stage nozzles, leaving all the subsequent passages thru the turbine unchanged in area, the pressure at every point in the turbine except the last few stages next the condenser remains very nearly proportional to the total flow of steam thru the turbine.

In our present example, therefore, the steam thrust will be reduced to $\frac{1}{7}$ while the propeller thrust is reduced to $\frac{1}{8}$, leaving an

unbalanced thrust forward of $\frac{1}{5}-\frac{1}{7}$, or $\frac{2}{35}$ of the whole propeller thrust at full power. A correction to be applied introduces an improvement. The condenser pressure while it falls somewhat, does not fall to $\frac{1}{7}$ of its value at full power, and as the condenser pressure is acting on the *forward* end of the L.P. rotor—in the case of the divided turbine—the steam thrust aft is in reality somewhat greater than $\frac{1}{7}$ and the balance closer than just computed.

The residual thrust can not, in any case, be computed very accurately, but it is quite evident that if the balance is good at full speed it can not be far off at half speed. The automatic readjustment of the pressure distribution which occurs upon a change in the amount of steam supplied, is favorable to preserving a balance existing before the change, and intermediate valves are superfluous in this respect as they have proved to be in others.

A more detailed treatment of this subject, with numerical illustrations, will be published elsewhere.

ABSTRACTS

Authors of scientific papers are requested to see that abstracts, preferably prepared and signed by themselves, are forwarded promptly to the editors. Each of the scientific bureaus in Washington has a representative authorized to forward such material to this journal and abstracts of official publications should be transmitted through the representative of the bureau in which they originate. The abstracts should conform in length and general style to those appearing in this issue.

ASTRONOMY.—*Publications of the U. S. Naval Observatory*, second series, **6**: pp. LXXIX + 620, illustrated, 10 plates, 4°. 1911.

When the Naval Observatory was moved in 1893 from the old site to the new one on Georgetown Heights it was decided to discontinue the practice of issuing an annual volume, and to publish volumes only occasionally, these to contain, however, rather large pieces of work. Following this practice Volume VI includes all the Equatorial Observations made during the period 1893–1907, inclusive, with the 26-inch and 12-inch instruments, together with three appendices. The observations of Eros for parallax, however, have already been published in Volume III (second series).

When the 26-inch was set up at the new site it was provided with a mounting and accessories furnished by Warner and Swasey, of the same general type as those of the large Lick and Yerkes telescopes. Of the old Clark instrument there remained only the object glass in its cell, the micrometers, the finders, and a few other small parts.

The walls of the 26-inch Equatorial building are of brick faced with rough white marble. In the plans for the new observatory, however, it was not intended that the dome of this instrument should have a masonry drum. It was proposed to erect steel uprights to carry the track for the dome, and have these uprights support a double wall of sheet metal which could be easily ventilated. As constructed the 26-inch telescope is very conveniently handled, and is stable. In these respects it excels, as far as can be learned, the large instruments made abroad.

As to some of the defects of the mounting, it may be noticed that although the driving clock is big and heavy, and furnishes plenty of power to carry the instrument, it has always had a troublesome periodic

error, the magnitude of which has been considerably reduced at the Observatory.

When the telescope is above the pier the thrust of the declination spindle is taken up by ball bearings, but when below the weight comes against a tool-steel collar evidently not intended to support it. It is clear that the designer did not quite comprehend how the instrument was to be used, since in the case of many objects in the south it is necessary to observe with the telescope under the pier.

When the 26-inch instrument left the makers the arrangement for illuminating the declination circle was not altogether satisfactory, and the circle could not be properly utilized. The illumination has been improved somewhat.

Three micrometers have been employed with the large Equatorial: Clark Micrometer I, Clark Micrometer II, and that by Warner and Swasey. The last has not been of much use. All these instruments have excellent screws, but were all constructed by men who were not quite familiar with the measurements that had to be made with them, and about the necessity for avoiding systematic errors, and they all have unnecessary defects.

In 1904 a bright field illumination was arranged by reflecting light from a small 2 candle power lamp to the inner surface of the objective by means of a prism within the telescope tube near the eye-end and just outside the cone of rays. A good illumination is furnished for magnifying powers up to about 550.

In the selection of work for the equatorial instruments the usual policy of the Naval Observatory has been followed, that observations should for the most part be for the improvement of the places of the Sun, Moon, and planets, and the fixed stars, and for the testing of the laws according to which the heavenly bodies are supposed to move. Usually, also, heavy pieces of observing are taken up which would not be cared for elsewhere, which it is necessary to extend over considerable periods, and in which it is important to secure continuity.

The plan of work, therefore, was to observe, (1) the satellites and the diameters of planets, (2) some of the interesting and difficult double stars, (3) minor planets and comets, and (4) occultations of stars by the Moon. The 26-inch has been devoted to the fainter and more difficult objects. Continuity in the observations of the satellites of Saturn, Uranus, and Neptune was considered important.

For about two years, 1898–1900, attempts were made to do spectroscopic work with the 26-inch Equatorial, in the way of determining

the motions of certain stars in the line of sight. However, on account of the strong colors existing in the flint and crown glasses of the objective no better results could be obtained than those given by a very much smaller instrument. It seemed proper, therefore, to discontinue spectroscopic investigations.

As to the observations included in this volume there are then: The satellites of Mars, measured with respect to the planet at five oppositions. These moons were discovered at the Naval Observatory, and their orbits were first determined there. They are interesting when any questions are considered relating to the evolution of the solar system, since they revolve about Mars in $7^h\ 39^m$ and $30^h\ 18^m$, respectively, while Mars itself rotates on its axis in $24^h\ 37^m$. Also, there are some very curious perturbations of the elements of these satellites, caused by the flattening of Mars.

The four bright moons of Jupiter were observed in pairs, according to the method of Otto Struve, during four oppositions. But since the bright moons can probably be measured more accurately in this way with the heliometer than with the equatorial the work was discontinued. Satellite VI, however, was observed with reference to fixed stars during the opposition of 1905–06, and has been measured regularly in that way since that time.

The satellites of Saturn were observed in pairs, using at first fifteen combinations, which number was afterwards reduced to nine. At one opposition, 1895, Hyperion was referred to Saturn.

The four moons of Uranus were referred directly to the planet, and the combination Titania-Oberon was also made. The satellite of Neptune was followed at every opposition except one.

These satellite measures are a proper extension of the work accomplished at the old Observatory site, which included rather elaborate monographs on the moons of Mars, Saturn, Uranus and Neptune, as well as the discovery of the very curious motion of the line of apsides of the orbit of Hyperion. The movement of the node and inclination of the orbit of Neptune's satellite, probably arising from a flattening of the planet, was pointed out by Mr. Marth as a result of the comparison of the Malta and Washington observations.

For the measures of diameters during the years 1900, 1901, and 1902 color cells were used to absorb the outstanding blue light of the secondary spectrum, these cells being placed in front of the eye-pieces.

A list was observed of 593 double stars. It was not intended to attempt any thing in the way of finding new doubles, but only to furnish needed measures of known pairs.

A list of 245 asteroids and 45 comets was observed. Many asteroids were secured at more than one opposition. A good many were found by means of photographic plates exposed for the purpose, and two were discovered at the Observatory. For some time the asteroids discovered by the Rev. J. H. Metcalf of Taunton, Mass., were observed systematically, for the purpose of securing sufficient data so that orbits could be computed, and the asteroids found and identified at the following oppositions. In the list are 20 of these bodies.

Considerable attention was given, also to the Watson asteroids. In general, it was intended to make observations of the minor planets only when measures had been lacking for some time.

Many of the observations contained in this volume have been printed in the *Astronomical Journal* and the *Astronomische Nachrichten*. Also, a good many discussions of observations have appeared in the same journals.

There are three appendices to Volume VI which include: the Mass of Titan from Observations of Hyperion; the Orbits of Phobos and Deimos; the Orbit of Enceladus; a Determination of the Solar Parallax from Observations of Eros; Orbits of Asteroids and Comets; Reports on the Transit of Mercury of November 10, 1894; and a List of Publications issued by the United States Naval Observatory, 1845–1908.

The introduction contains convenient summaries, so that the material contained in the volume proper can be found easily. ASAPH HALL.

TERRESTRIAL MAGNETISM.—*Two new types of magnetometers made by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington.* J. A. FLEMING. Journal of Terrestrial Magnetism and Atmospheric Electricity, **16**: 1–12, pls. I and II. 1911.

The instruments described are the result of the experience gained in the land operations of the Department of Terrestrial Magnetism, which has emphasized the great need of more portable and more compact instrumental outfits than were heretofore in general use, without any sacrifice in the degree of precision obtainable. The controlling conditions in the designs and construction of the instruments described were: (a) portability; (b) compactness; (c) simplicity; (d) minimum of loose accessories; (e) readiness for immediate use; (f) the attainment of an absolute observational accuracy equal to that of the best field instruments now in use.

The new instruments are of two types. The first is a theodolite mag-

netometer for astronomical work and the determination of magnetic declination and horizontal intensity; in general principles along usual lines but differing greatly in details. The second type is a universal magnetometer by means of which the three magnetic elements may be determined as well as the astronomical elements.

The magnet and suspension systems of both types are of like design. The magnets are true cylinders encased in gold-plated brass cylindrical sheaths to prevent rusting and to provide means of mounting the collimating optical system in order that the magnets may be left as perfect cylinders. The suspension systems are comparatively free from complex or intricate parts, all devices for clamping the phosphor-bronze ribbon used for suspension being such that they are not removable and may be manipulated with great ease. For the determination of the plane of no torsion there is attached permanently to the stirrup a graduated circle which may be brought into the focus of the magnetometer telescope (previously focused on a distant mark) by interposing an auxiliary lens; the graduations of the torsion circle are in divisions of two degrees so that the plane of detorsion may be rapidly determined within a fraction of a degree.

The theodolite magnetometer weighs with its case but 11 kilograms, which is about half that of the older forms of corresponding instrument used by this Department.

The universal magnetometer is completely assembled in one unit, comprising astronomical telescope and magnetometer for determination of magnetic declination and horizontal intensity, and dip circle for the determination of inclination and total intensity, and is therefore always ready for immediate use upon mounting on its tripod. The advantage of being able to proceed with the observations for any of the elements, without first assembling a number of parts, is evident and of peculiar value in taking advantage of every available opportunity for work. The weight of this instrument without its packing case is $7\frac{1}{4}$ kilograms; the case weighs about 6 kilograms so that the total weight is only about 13 kilograms.

The results of comparisons with the new instruments, both at the office and under field conditions, have clearly shown that the degree of precision obtainable with them is of the same order as that to be had by types of standard instruments previously used by the Department and other magnetic organizations, or, with care, an absolute accuracy of 0.5 minute in declination, 0.0002H in horizontal intensity, and about one-half minute in inclination. J. A. F.

BIOLOGICAL CHEMISTRY.—*Further experiments upon the determination of iodine in thyroid.* ATHERTON SEIDELL, Hygienic Laboratory, Public Health and Marine Hospital Service. Journal of Biological Chemistry, **10**: 95–108, Oct. 1911.

In this paper are presented the results of an extensive series of analyses made for the purpose of comparing the Hunter (J. Biol. Chem., **7**: 321–49. 1910), and Baumann methods for the determination of iodine in thyroid. The results demonstrate the superiority of the Hunter method, both as to reliability of results and simplicity of manipulation. Further evidence disproving the contention of Riggs that iodate is formed during the fusion of thyroid by the Baumann process is presented. In practically all cases it was found that the older Baumann method gives only 80 to 90 per cent of the amount of iodine obtained by the Hunter method. The most probable cause of this difference is considered to be the loss of iodine which occurs during the acidification of the aqueous solution of the fused residue, during which there is an active evolution of CO_2 and since some of the nitrate of the fusion mixture is always reduced to nitrite, there must also be present free iodine, which is liberated by the nitrous acid generated from the nitrite upon acidification. This source of loss has been effectively overcome in the Hunter method, by converting the inorganic iodine to the iodic state, and at the same time removing the nitrite by means of sodium hypochlorite solution, before the aqueous solution is acidified. Duplicate determinations by the Hunter method are therefore quite satisfactory, while in the case of the Baumann method frequent irregularities of results which can only be explained on the assumption of a loss, are obtained.

Since it has been claimed by Koch (Proc. Am. Pharm. Assoc., **55**: 371. 1907), that the iodine content of desiccated sheep thyroids prepared at different seasons of the year was as much as three times as great in the winter as in the summer months, an experiment was made for the purpose of demonstrating the possible seasonal variation of the iodine content of dog thyroids. The glands from groups of dogs killed at intervals during more than a year at the Washington dog pound were analyzed for their iodine, but no regular variation with season could be detected. It appeared that the cause of this irregularity might be the individual variation of the dogs, and analyses were therefore made of the dried glands of individual dogs, with the result that very wide differences (0.036 to 0.271 per cent iodine) were found. From these results it was concluded that if there is a seasonal variation in the iodine content of the thyroid of dogs, the individual variations under ordinary conditions

are far too great to permit its detection by analyses of the collected glands from a reasonable number of dogs.

Thru the kindness of the firms of Armour and Company, and Parke, Davis and Company, samples of the several lots of their U. S. P. desiccated sheep thyroids manufactured during the past year and a half were sent to this laboratory for use in the work upon the pharmacopœial standards for this drug. The dates of the preparation of the several lots were given, and these together with the analytical results, furnished the necessary data upon the question of the seasonal variation of the iodine content. It was found that the results did not confirm the observation of Koch; there being the greatest possible irregularity with season. It cannot be claimed, however, that these results disprove Koch's claim since the glands from which the present lots of desiccated thyroid were prepared came from both sheep and lambs, and this fact is probably sufficient to explain the variation. It may be concluded, however, that unless Koch exercised particular care to have only the glands from sheep in his average lots, his results must have been due to an accidental regularity. A. S.

PHARMACOLOGY—*Influence of diet on the thyroid gland.* REID HUNT, Hygienic Laboratory. Journal American Medical Association, **57**: 1032. 1911.

Indirect evidence had been obtained (Bulletin 69, Hygienic Laboratory) that certain diets have specific effects upon the thyroid gland of animals; in the present series of experiments direct evidence for this was obtained. After removal of the thyroid certain diets no longer produced certain characteristic effects; the thyroid glands themselves showed marked physiological effects resulting from the diets. Oatmeal and liver were found to increase the activity of the glands; milk and eggs decreased it. The latter diet markedly increased the reproductive activity of the animals (mice and rats) used in the experiments. R. H.

GEOLOGY.—*Foxburg-Clarion, Pa., Folio.* E. W. SHAW and M. J. MUNN. Geologic Atlas of the United States, No. 178. Pp. 17, with maps and sections. U. S. Geological Survey. 1911. *Coal, oil and gas of Foxburg Quadrangle, Pa.* E. W. SHAW and M. J. MUNN. Bulletin U. S. Geological Survey No. 454. Pp. 85, with maps and sections. 1911.

Outcropping rocks, aside from the Quaternary stream deposits, belong to the Cuyahoga, Burgoon, Allegheny, and Conemaugh forma-

tions of the Carboniferous. The lowest rock (Cuyahoga shale) appears in the bottom of the Allegheny Gorge; the Burgoon and Pottsville sandstones outcrop along the valley sides, and the Allegheny and Conemaugh, which consist of shale, sandstone, and limestone, with interbedded coal and clay, appear higher in the hills. The dip is gently to the south with modification by low folds.

The principal streams are Allegheny and Clarion rivers and Redbank Creek, and along these a much discussed system of high terraces and abandoned valleys is well developed.

The coal beds lie mainly in the Allegheny formation. The lower and upper Clarion, lower Kittanning, and upper Freeport coals are the most important. Oil and gas in great quantities have been found in sandstones of the upper part of the Devonian and lower part of the Carboniferous. E. W. S.

GEOLOGY.—*Geology and mineral resources of the St. Louis Quadrangle, Mo.-Ill.* N. M. FENNEMAN. Bulletin U. S. Geological Survey No. 438. Pp. 73, with maps, sections, and views. 1911.

The hard rocks outcropping in the St. Louis quadrangle are all of Carboniferous age and include the Osage, Merrimac, and Chester (?) groups of the Mississippian, all of which have been further subdivided, and the Pennsylvanian coal measures. Older rocks, including the Kinderhook formation (Mississippian) and part of the upper, Ordovician have been reached by drilling. The Paleozoic rocks are unconformably overlain by Lafayette gravel of Tertiary age. The Pleistocene is represented by the Kansan and Illinoian drifts, by loess, and by glacial outwash of the Wisconsin stage. The recent deposits of the region include terrace gravels and the alluvial filling of river valleys.

The Paleozoic strata dip in general north-northeasterly, the fall being from 50 to 60 feet to the mile. A. H. BROOKS.

PROGRAMS AND ANNOUNCEMENTS

PHILOSOPHICAL SOCIETY

699th Meeting, November 11, 1911, Cosmos Club at 8.15.

A. J. LOTKA: *Evolutions in discontinuous systems.*